**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. nd = norm(55, 8)

nd.cdf(60)

failure = 1-nd.cdf(60)

print(failure)

**Output :** 0.2659

Therefore, the Probability that the service manager cannot meet his commitment is 0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

**A. More employees at the processing center are older than 44 than between 38 and 44.**

Ans : import numpy as np

from scipy.stats import norm

nd1 = norm(38, 6)

print("P(X>44) =", np.round(1-nd1.cdf(44),3)\*100,"%")

print("P(38,X<44) =", np.round(nd1.cdf(44) - nd1.cdf(38),4)\*100,"%")

**Output :**

P(X>44) = 15.9 %

P(38<X<44) = 34.13 %

The **Statement A** (more employees at the processing center are older than 44 than between 38 and 44) is **False,** because the probability of no of employees at the processing center older than the age 44 is (15.9%) while the no of employees in the age between 38 and 44 is (34.13%)

**B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Ans : import numpy as np

from scipy.stats import norm

nd1 = norm(38, 6)

print("P(X<30) =", np.round(nd1.cdf(30),2)\*100,"%")

print("A training program for employees under the age of 30 at the center would be expected to attract about",((9.0\*400)/100),"employees")

**Output :**

P(X<30) = 9.0 %

A training program for employees under the age of 30 at the center would be expected to attract about 36.0 employees

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
2. **Distribution of 2X1 :**

If X1 follows a normal distribution with mean(μ) and variance(σ2) , then 2X1 will follow a normal distribution with following mean and variance

Mean : E[2X1] = 2E[X1] = 2μ

Variance : Var(2X1) = 22Var(X1) = 4σ2

So the distribution of 2X1 is N(2μ, 4σ2)

1. **Distribution of X1 + X2 :**

The sum of independent normal random variables is also a normal random variable. The distribution of the sum X1 + X2 is given by:

Mean: E[X1 + X2] = E[X1] + E[X2] = μ + μ = 2μ

Variance: Var(X1 + X2) = Var(X1) + Var(X2) = σ2 + σ2 = σ2

***2X1 ~ N(2μ, 4σ2)***

***X1 + X2 ~ N(2μ, 2σ2)Top of Form***

(1) Both 2X1​ and X1​+X2​ are normal random variables.

(2) The means of 2X1 and X1 + X2 are equal.

(3) However, 2X1​ ​ has a higher variance compared to X1​+X2​. This is because the factor of 2 in 2X1​ ​ contributes to a larger variance compared to the sum of two variables in X1​+X2​ ​.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

A) from scipy import stats

from scipy.stats import norm

import numpy as np

print(np.round(stats.norm.interval(0.99, loc = 100, scale = 20),1))

**Output :**

[ 48.5, 151.5]

Therefore, probability of the random variable taking a value between them is 0.99: [ 48.5 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

A) import numpy as np

from scipy import stats

from scipy.stats import norm

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

**Output :**

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

**B. Specify the 5th percentile of profit (in Rupees) for the company**

A) X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

**Output :**

5th percentile of profit (in Million Rupees) is 170.0

**C) Which of the two divisions has a larger probability of making a loss in a given year?**

A) The Division 2 has a larger probability of making a loss in a given year